Math 4 Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**5-7 Optimization Problems** Date \_\_\_\_\_\_\_\_\_\_

* *How can derivatives be used to optimize a quantity?*

**The Process:**

 1. Determine whether it is a maximum or minimum problem.

 2. Determine what has to be maximized or minimized.

 3. Select variables to represent unknowns & write the equation of a function to represent the problem.

 4. Use the first derivative of the function to find the critical points.

 5. Justify why your solution is the maximum or a minimum.

 6. Use the critical point(s) to answer the question.

\*\*\*\*You will need to do the examples & homework on your own paper.\*\*\*\*

**Examples Day 1:**

 1. Find two numbers whose sum is 18 and whose product is the greatest possible.

 2. The square of a number is to be added to twice the number. What number will give the smallest sum?

 3. A rectangular field is to be enclosed using 200 feet of fencing. Find the dimensions of the field of maximum area.

**Homework Day 1:**

 1. If the square of a number is to be subtracted from twice the number, find the number that will give the greatest difference.

 2. Find the number which when added to the square of the number will yield the least sum.

 3. If the square of a number is to be subtracted from the number, determine the number which will give the greatest difference.

 4. Find two numbers whose difference is 10 and whose product is the minimum possible.

 5. A rectangular field is to be enclosed by using 100 feet of fencing. Find the dimensions of the field of maximum area. What is the maximum area?

 6. A rectangular field is to be enclosed using 100 feet of fencing using a river bank as one side. Find the dimensions of the field of maximum area.



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 **Examples Day 2:**

 1. A box is made by cutting equal squares out of the corners of a rectangular piece of cardboard 6" by 16" and folding up the flaps. Find the dimensions of the box which will have maximum volume.

 2. Determine the radius and height of the cylinder of maximum volume that can be obtained by revolving a rectangle of perimeter 24" about one of its sides.

 \*3. On the graph *y* = *x*3 take a point **P**(*x*, *y*) subject to the condition 0  *x*  6. Join point P to the point (6, 0) by a straight line and drop a perpendicular from P to the *x*-axis. These two lines and the *x*-axis form a right triangle. Find the value of *x* for which the area of the triangle is a maximum.

 \*4. A grower estimates that if the crop of oranges is harvested now, the average yield of 80 pounds per tree can be sold at $.40 per pound. From past experience, the owner expects the crop yield to increase at a rate of 10 pounds per week per tree, and the price to decrease at a rate of $.02 per pound per week. When should the oranges be picked in order to attain maximum sale?

 **Homework Day 2:**

 1. The cube of a non-negative number is subtracted from the square of the same number. Find the number, which gives the greatest difference.

 \*2. The sum of two positive numbers is 20. If P is the product of one number and the square of the other, find the two numbers that will maximize P.

 \*3. Find two positive numbers whose product is 64 and whose sum is as small as possible.

 4. Find the number which when added to twice the square of the number will give the least sum.

 5. Find two numbers whose difference is 22 and whose product is a minimum.

 6. Three times the square of a number is subtracted from the cube of the number. If the number is non-negative, find the number that gives the least difference.

 7. Cutting equal squares out of the corners of a piece of cardboard 12” by 12” and folding up the flaps makes a box. Find the dimensions of the box which will have maximum volume.



\* **= challenge problem**

**Homework: Day 3** \* **= challenge problem**

 \*1. A long rectangular sheet of metal 12” wide is to be made into a rain gutter by turning up two sides at right angles to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

 \*2. A right triangle is revolved about one of its legs to form a cone. If the hypotenuse is, what should the dimensions of the cone be in order to obtain the maximum volume?

 3. A woman wants to design a rectangular garden with an ornamental fence around it. The fencing for three sides of the garden costs $2 per foot, the fencing for the 4th side costs $3 per foot. It she has $120 to spend, what dimensions should she give the garden to maximize the area?

 4. Cutting equal squares out of the corners of a piece of cardboard 15” by 8” and folding up the flaps makes a box. Find the dimensions of the box which will have maximum volume.

 5. A man has 180 feet of fencing to enclose a rectangular field. If a river bank is going to be used as one of the sides and he also wishes to subdivide the field with a fence perpendicular to the river bank, what are the dimensions of the field of maximum area?

 \*6. A realtor wishes to enclose 600 square meters of land in a rectangular plot and then subdivide it into two plots with a fence parallel to one of the sides. What are the dimensions of the rectangle that will use the least fencing?

 \*7. Design an open rectangular box with square ends having a volume of 36 cubic inches that minimizes the amount of material required for construction.

 \*8. A cylindrical can with no top is to be made so its volume is cubic inches. What dimensions should it have to minimize the material needed to make it?

 \*9. A right triangle is revolved about one of its legs to form a cone. If the hypotenuse is, what should the dimensions of the cone be in order to obtain the maximum volume?

 \*10. A right triangle is revolved about one of its legs to form a cone. If the sum of the lengths of the legs is 15”, what is the maximum volume?

 \*11. A rectangle of perimeter 24” is rotated about one of its sides to generate a cylinder. What should the radius and height of the cylinder be in order to maximize the volume?

 **OVER 🡪**

 \*12. Of all the rectangles with base on the *x*-axis, and upper corners on the parabola, what are the dimensions of the rectangle with largest area? What is the largest area?

 \*13. A cylindrical reservoir, open at the top, with a capacity of cubic feet, is to be constructed using sheet steel. What dimensions for the reservoir should be chosen to minimize the amount of sheet steel required?

 \*14. A rectangular sheet of paper with total area of 100 sq. inches is to be used to for a menu. The menu is printed with 2 inch margins at the top and bottom and 1 inch margins at the sides. For what dimensions of the sheet of paper is the printed area the largest?

 \*15. A right circular cylinder can with top is to be made from square inches of material. What are the dimensions of the can with maximum volume?

 \*16. A rectangular box with a square bottom is to have a fixed volume of 375 cubic inches. The top and bottom are to be padded with form and press-board which costs 3 cents per square inch, while the sides are made of fiber-board, which costs 1 cent per square inch. What dimensions produce the box of least cost?

 \*17. A canvas wind shelter for the beach has a back, two square sides, and a top. Suppose that 96 square feet of canvas are to be used. Find the dimensions of the shelter for which the space inside the shelter will be maximized.

 \*18. The manager of a department store wants to build a 600 square foot rectangular enclosure on the store’s parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing at a cost of $7 a running foot. The 4th side will be built of cement block at a cost of $14 a running foot. Find the dimensions of the enclosure, which minimizes the total cost of the building materials.

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